Government College of Engineering and Research, Avasari(Khurd)

Department: Mechanical Engineering

Learning Resource Material (LRM)

Name of the course: Mechanical System Design **Course Code:** 402048

Name of the faculty: J. M. Arackal **Class:** BE(Mech)

SYLLABUS (Unit 2)

Unit 2: Statistical considerations in design (6 Hours)

Frequency distribution-Histogram and frequency polygon, normal distribution - units of of central tendency and dispersion- standard deviation - population combinations - design for natural tolerances design for assembly - statistical analysis of tolerances, mechanical reliability and factor of safety.

Lecture Plan format:

Name of the course: Mechanical System Design **Course Code** 402048

Name of the faculty: J. M. Arackal Class: BE(Mech)

Unit No	Lecture No.	Topics to be covered	Text/Reference Book/ Web Reference
		UNIT 2	
2	1	Frequency distribution-Histogram and frequency	1,2
2	2	units of of central tendency and dispersion	1,2
2	3	standard deviation - population combinations - design for natural tolerances	1,2
2	4	Problems on Basic Terms	1,2
2	5	Problems on Statistics used for Assembly of Parts	1,2
2	6	Problems on use of Statistics in Reliability	1,2

List of Text Books /Reference Books/ Web Reference

1-Bhandari V.B. ―Design of Machine Elements‖, Tata McGraw Hill Pub. Co. Ltd.

2-R.K. Jain- Machine Design, Khanna Publishers

3-Johnson R.C., ―Mechanical Design Synthesis with Optimization Applications‖, Von Nostrand Reynold Pub

Statistical Considerations in Design.

Statistics deals with drawing conclusion from a given or observed data.

Statistical techniques are used for collection, processing, analysis & interpretation of numerical data.

Statistics has made valuable contributions in the area of product design. and manufacture & effective use of material & labour.

Basic data consists of observations. such as.

i) dia of the shaft manufactured in one shift.

population is defined as a collection of ~~some~~ all elements we are studying and about which we are trying to draw conclusions.

Sample is defined a collection of some but not all, of the elements of the population.

sample is a part of population. A representative sample has the characteristics of the population in the same proportions, as they are included in the entire population.

A data is defined as. the collection of numbers belonging to observations of one or more variables

39.944	39.935	39.946	39.946	39.939
39.932	39.937	39.941	39.940	39.941
39.938	39.940	39.932	39.943	39.929
39.934	39.941	39.936	39.939	39.939
39.939	39.938	39.937	39.933	39.943

Dia	no of shafts	Total
39.929	X	1
39.932	XX	2
39.933	X	1
39.934	X	1
39.935	X	1
39.936	X	1
39.937	XX	2
39.938	XX	2
39.939	XXXX	4
39.940	XX	2
39.941	XXX	3
39.943	XX	2
39.944	X	1
39.946	XX	2

Table : Dia ~~of~~ no of Shafts.

frequency distribution.

shaft dia		No of shafts.
39.928 – 39.932	————	3 ← frequency.
39.933 – 39.937	————	6.
39.938 – 39.942	————	11
39.943 – 39.947	————	5

classes

~~The number of observation belonging to each c~~

frequency distribution is defined as an organised display of data that shows the number of observations that fall in different classes

The no of observations belonging to each class. is called class frequency.

~~The dif~~ The range 39.928 to 39.932 mm which defines the class, is called class interval.

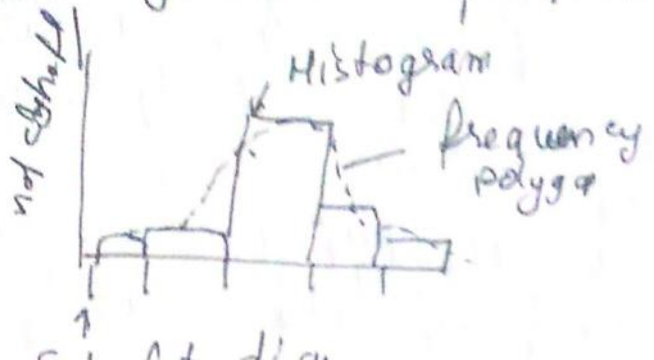
& the limits, upper & lower limits.

The difference between limits is called class width.

Equal classes are preffered in statistical analysis

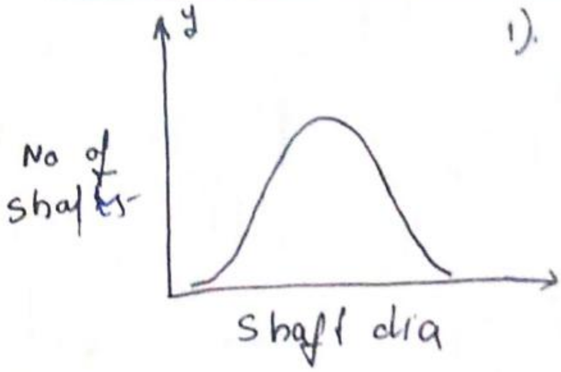
There are two methods of representing frequency distribution.

histogram & frequency polygon.



frequency polygon is a line graph of class frequency plotted against class marks or midpoints of class Intervals.

Characteristics of frequency curves



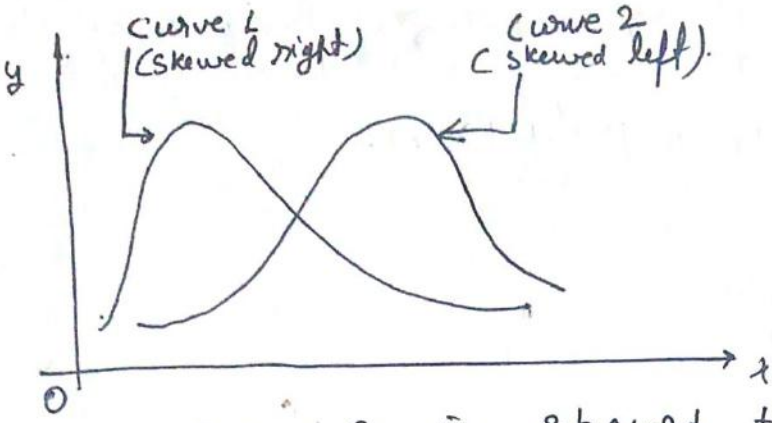
1). Central tendency : Its the middle point of distribution. Its also referred as measure of location.

- ~~CT~~ CT of curve 1 & curve 2 are same.

- Central location of curve 3 is to the right of curve 1 & 2

2) Variation / Dispersion : Its defined as the spread of the data in a distribution, that is the extent to which the observations are scattered.

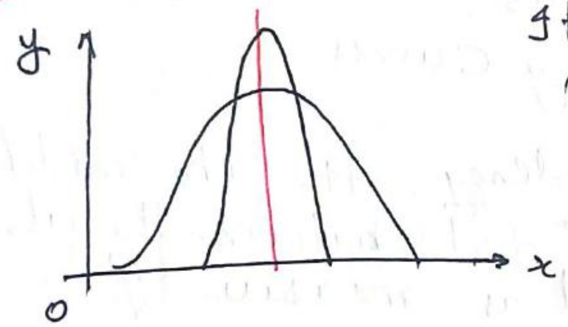
3) Skewness : In skewed curves, the values in frequency distribution are concentrated at either the low end or the high end of the measuring scale on horizontal axis.



Curve 1 is skewed to right because it tails off towards the high end of the scale. Its also called positively skewed curve.

Curve 2 is skewed to the left because it tails off toward the low end of the scale. Its called negatively skewed curve.

Kurtosis.



Its the measure of sharp peak.

Measure of Central tendency & Dispersion.

There are different measures of central tendency mean, median or the mode. The most popular and to measure is the arithmetic mean denoted by μ. let a population of N have observations $x_1\ x_2 \ldots x_N$..

$$\mu = \frac{x_1 + x_2 + \ldots x_N}{N}$$

$$\mu = \frac{\Sigma x_i}{N}$$

If observations $x_1\ x_2 \ldots x_k$ occur $f_1\ f_2 \ldots f_k$

$$\therefore \mu = \frac{f_1 x_1 + f_2 x_2 + f_3 x_3 + \ldots f_k x_k}{f_1 + f_2 + \ldots f_k}$$

$$\mu = \frac{\Sigma f_i x_i}{\Sigma f_i} \qquad \therefore \boxed{\mu = \frac{\Sigma f_i x_i}{N}}$$

Dispersion : Its measured in numbers of units like. the range, the mean deviation or the standard deviation standard deviation ($\hat{\sigma}$) is the most popular unit.

standard deviation is defined as the root mean square deviation from the mean.

$$\hat{\sigma} = \sqrt{\frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \ldots (x_N - \mu)^2}{N}}$$

$$\hat{\sigma} = \sqrt{\frac{\Sigma (x_i - \mu)^2}{N}}$$

$$\hat{\sigma} = \sqrt{\frac{f_1 (x_1 - \mu)^2 + f_2 (x_2 - \mu)^2 + \ldots f_k (x_k - \mu)^2}{f_1 + f_2 + \ldots f_k}}$$

$$\hat{\sigma} = \sqrt{\frac{f_i (x_i - \mu)^2}{\Sigma f_i}} \qquad \therefore \boxed{\sigma = \sqrt{\frac{\Sigma f_i (x_i - \mu)^2}{N}}}$$

squaring on b-s we have.

$$\hat{\sigma}^2 = \frac{1}{N} \Sigma f_i (x_i - \mu)^2.$$

$$(\hat{\sigma})^2 = \frac{1}{N} \Sigma f_i(x_i^2 - 2x_i\mu + \mu^2)$$

$$= \frac{1}{N} \Sigma f_i x_i^2 - 2\mu \frac{\Sigma f_i x_i}{N} + \mu^2 \frac{\Sigma f_i}{N}$$

$$= \frac{\Sigma f_i x_i^2}{N} - 2\mu^2 + \mu^2 \quad \left\{ \because \frac{\Sigma f_i x_i}{N} = \mu \ \& \ \Sigma f_i = N \right.$$

$$(\hat{\sigma})^2 = \frac{\Sigma f_i x_i^2}{N} - \frac{(\Sigma f_i x_i)^2}{N^2}.$$

$$\cancel{(\hat{\sigma}) = \frac{\Sigma f_i x_i^2 - (\Sigma f_i x_i)^2/N}{N^2}}$$

$$\boxed{(\hat{\sigma})^2 = \frac{\Sigma f_i x_i^2 - \frac{(\Sigma f_i x_i)^2}{N}}{N}}$$

A better estimate of standard deviation is obtained when N is replaced by N − 1 for N > 30 [larger values of N].

∴ standard deviation

$$s^2 = \frac{\Sigma f_i x_i^2 - \frac{(\Sigma f_i x_i)^2}{N}}{(N-1)}.$$

where s is the standard deviation of observations belonging to the sample of the population.

variance is defined as the square of standard deviation.

A standard variable Z is defined as

$$Z = \frac{x - \mu}{\hat{\sigma}}$$ // A standard variable measures the deviation from the mean in the units of the standard deviation

) one hundred test specimens made of grey cast iron FG 300 are tested on a universal testing machine to determine the ultimate tensile strength (S_{ut}) of the material. The results are tabulated as follows.

Class Interval (N/mm²)	Frequency.
261 - 280.	2
281 - 300.	12
301 - 320	50
321 - 340	32
341 - 360	4.

Calculate i) The mean ii) The variance &
iii) the standard deviation of for this sample

A). $$s^2 = \frac{\Sigma f_i x_i^2 - \frac{(\Sigma f_i x_i)^2}{N}}{N-1.}$$

Class mark (x_i).	f_i	$f_i x_i^2$	$(f_i x_i)^2$.	$f_i x_i$
270	2.	145800	291600	540
290	12	1009200	12110400	3480
310	50	4,805,000		15,500
330	32	3,484,800		10560
350	4.	490,000		1400
Σ.	100.	9934800		~~30,934~~. 31480.

Variance $$s^2 = \frac{\Sigma f_i x_i^2 - \frac{(\Sigma f_i x_i)^2}{N}}{N-1.}$$

$$s^2 = \frac{9934800 - \frac{(31480)^2}{100}}{99.}$$

variance, $s^2 = 251.47 \cdot (N/mm^2)^2$

standard duration, $s = 15.85 \cdot N/mm^2$.

$$\mu = \frac{\Sigma f_i x_i}{N} = \frac{31480}{100} = 314.8\ N/mm^2.$$

Probability

Probability is defined as the chance or likelihood that a particular event will occur; it varies from 0 to 1.

It indicates the chana that at particular event E will occur, given that it can happen f ways out of n equally likely ways.

$$p = P(E) = \frac{f}{n}.$$

If the event does not occur, it is called not E and written as $\bar{E}$

$$q = P(\bar{E}) = 1 - P(E).$$

$$p + q = 1.$$

Q) Five bolts with internal cracks are accidently mixed with 95 bolts without any defects. what is the probability that the assembly shop will use a defective bolt? Also, find out the possibility of not using the defective bolts.

A) Here $f = 5$.
$n = 100$.

$$P = P(E) = \frac{5}{100} = 0.05.$$

$$\& \bar{E} = 1 - 0.05 = 0.95.$$

Probability distribution

The testing on a UTM. is called random variable experiment because specimens are selected at random. The values of UTS. obtained in such testing are called. random variable.

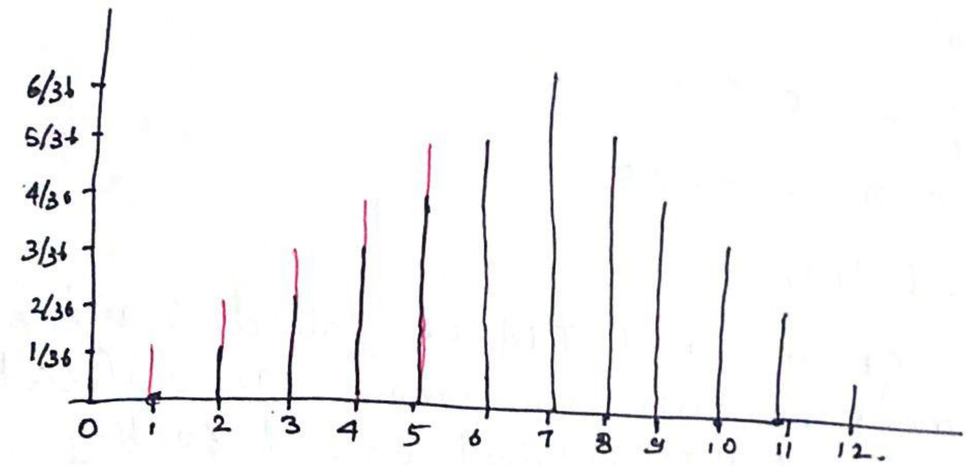
A random variable is defined as a variable that takes different values in random experiments.

Consider an expt of tossing two cubes.
Let variable be x. such that it denotes summation or addition of numbers

Possible out come

1 1	1 2	1 3	1 4	1 5	1 6
2 1	2 2	2 3	2 4	2 5	2 6
3 1	3 2	3 3	3 4	3 5	3 6
4 1	4 2	4 3	4 4	4 5	4 6
5 1	5 2	5 3	5 4	5 5	5 6
6 1	6 2	6 3	6 4	6 5	6 6

x	Number of events	Probability
1	0	0/36
2	1	1/36
3	2	2/36
4	3	3/36
5	4	4/36
6	5	5/36
7	6	6/36
8	5	5/36
9	4	4/36
10	3	3/36
11	2	2/36
12	1	1/36.



Probability distribution.

:- probability that x is less than 4.

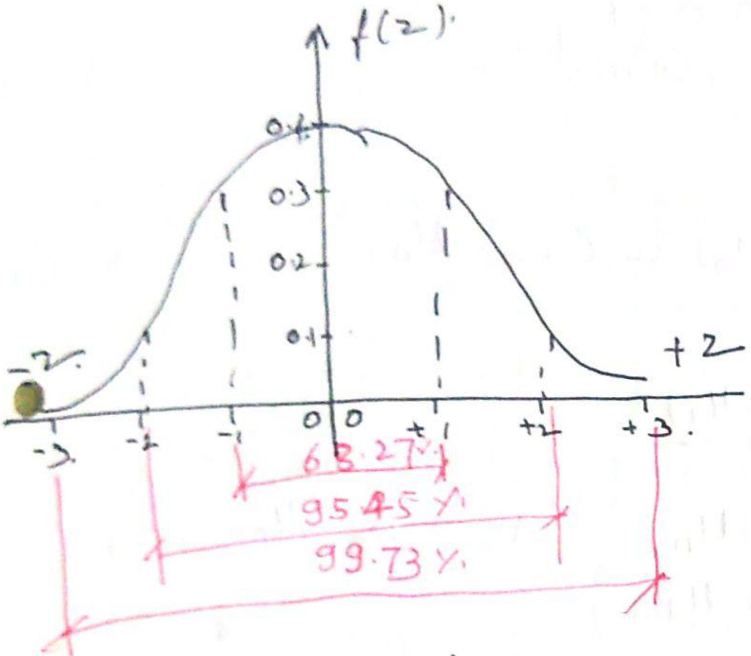
$$= 0 + \frac{1}{36} + \frac{2}{36} + \frac{3}{36} = \frac{6}{36}.$$

The probability that x is less than x_i is called cumulative probability

Normal curve

In statistical analysis, the most popular probability distribution curve is the normal curve, the distribution is also called Gaussian di.

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}.$$



The total area below the curve from $-\infty$ $z=-\infty$ to $z=+\infty$ is one or unity.

Population combinations-

There are many problems in machine design where its required to combine two or more populations in a specific manner to obtain resultant population.

Consider a simple case of three bearings with diameters D_1, D_2 & D_3 and two shaft diameters d_1 & d_2.

$$\therefore \mu_D = \frac{D_1 + D_2 + D_3}{3} \qquad \mu_d = \frac{d_1 + d_2}{2}$$

There are six possible combinations of shaft & bearings.

$$C_1 = D_1 - d_1 \qquad C_2 = D_2 - d_1 \qquad C_3 = D_3 - d_1$$

$$C_4 = D_2 - d_2 \qquad C_5 = D_3 - d_1 \qquad C_6 = D_3 - d_2.$$

$$\mu_c = \frac{(D_1-d_1)+(D_1-d_2)+(D_2-d_1)+(D_2-d_2)+(D_3-d_1)+(D_3-d_2)}{6}$$

$$= \frac{2(D_1+D_2+D_3) - 3(d_1+d_2)}{6}.$$

$\mu_D = \frac{D_1 + D_2 + D_3}{3}$ $\mu_d = \frac{d_1 + d_2}{2}$.

$\therefore \boxed{\mu_c = \mu_D - \mu_d}$ Therefore when two population are substracted, the mean of the resultant population is obtained by a substraction of their individual means.

$\Rightarrow \mu = \mu_x - \mu_y$. , similarly

$\mu = \mu_x + \mu_y$

we have.

$$(\hat{\sigma}_D)^2 = \frac{(D_1 - \mu_D)^2 + (D_2 - \mu_D)^2 + (D_3 - \mu_D)^2}{3}.$$

$$(\hat{\sigma}_d)^2 = \frac{(d_1 - \mu_d)^2 + (d_2 - \mu_d)^2}{2}$$

Let

$A_1 = D_1 - \mu_D$ $A_2 = D_2 - \mu_D$ $A_3 = D_3 - \mu_D$.

$B_1 = d_1 - \mu_d$ $B_2 = d_2 - \mu_d$.

$$(\hat{\sigma}_D)^2 = \frac{A_1^2 + A_2^2 + A_3^2}{3}. \qquad (\hat{\sigma}_d)^2 = \frac{B_1^2 + B_2^2}{2}.$$

$\therefore$ we have

$$(\hat{\sigma}_c)^2 = \frac{(D_1 - d_1 - \mu_c)^2 + (D_1 - d_2 - \mu_c)^2 + (D_2 - d_1 - \mu_c)^2 + (D_2 - d_2 - \mu_c)^2 + (D_3 - d_1 - \mu_c)^2 + (D_3 - d_2 - \mu_c)^2}{6}.$$

But $\mu_c = \mu_D - \mu_d$.

$$(\hat{\sigma}_c)^2 = \frac{(D_1 - d_1 - \mu_D + \mu_d)^2 + (D_1 - d_2 - \mu_D + \mu_d)^2 + (D_2 - d_1 - \mu_D + \mu_d)^2 + (D_2 - d_2 - \mu_D + \mu_d)^2 + (D_3 - d_1 - \mu_D + \mu_d)^2 + (D_3 - d_2 - \mu_D + \mu_d)^2}{6}.$$

$$= \frac{(A_1 - B_1)^2 + (A_1 - B_2)^2 + (A_2 - B_1)^2 + (A_2 - B_2)^2 + (A_3 - B_1)^2 + (A_3 - B_2)^2}{6}$$

$)^2 = A_1^2 - 2A_1B_1 + B_1^2 + A_1^2 - 2A_1B_2 + B_2^2 + A_2^2 - 2A_2B_1 + B_1^2 +$

$$\frac{A_2^2 - 2A_2B_2 + B_2^2 + A_3^2 - 2A_3B_1 + B_1^2 + A_3^2 - 2A_3B_2 + B_2^2}{6}$$

$$= \frac{2(A_1^2 + A_2^2 + A_3^2) + 3(B_1^2 + B_2^2) - 2[A_1B_1 + A_1B_2 + A_2B_1 + A_2B_2 + A_3B_1 + A_3B_2]}{6}$$

$$= \frac{2(A_1^2 + A_2^2 + A_3^2) + 3(B_1^2 + B_2^2) - 2[(A_1 + A_2 + A_3)B_1 + (A_1 + A_2 + A_3)B_2]}{6}$$

$$= \frac{2(A_1^2 + A_2^2 + A_3^2) + 3(B_1^2 + B_2^2) - 2[(A_1 + A_2 + A_3)(B_1 + B_2)]}{6}$$

we have $A_1 = D_1 - \mu_D$.

$\therefore A_1 + A_2 + A_3 = D_1 + D_2 + D_3 - 3\mu_D = 0$ $\left[\because \mu_D = \frac{D_1 + D_2 + D_3}{3}\right]$

$$\therefore (\hat{\sigma}_c)^2 = \frac{A_1^2 + A_2^2 + A_3^2}{3} + \frac{B_1^2 + B_2^2}{2}.$$

$$(\hat{\sigma}_c)^2 = (\hat{\sigma}_D)^2 + (\hat{\sigma}_d)^2.$$

$\therefore$ standard. deviation follows the pythagorean rule

The above is valid for an addition of two populations.

In statistical analysis, many a times its important to know the distribution of the resultant population obtained by combination of two or more population.

1) when two normally distributed random variables are added, the resulting population is also normally distributed

2) when two normally distributed random variables are substracted, the resulting population is also normally distributed.

3) when two normally distributed random variables are multiplied, the resulting population has approximately normal distribution.

iv) when two normally distributed random variables are divided, the resulting population has not strictly normally distribution, It can be approxim normal distribution.

Design & Natural Tolerance

The variation in the dimensions of a compo occur due to two reasons.

1- Because of large number of chance causes.

2- Assignable Causes.

Chance causes occur at random, they ar characteristics of the manufacturing method & measurement technique.

The variations due to assignable causes. can be located & corrected, the system is said to be under statistical control.

In a statistically controlled system, the dimensions of the component are normally distributed with a particular value of valie. standard deviation.

The natural tolerance is defined as the actual capabilities of the process, and can be considered as limits within which all but a given allowable fraction of items will fall.

In general natural tolerance of a process is the spread of the normal curve that includes 99.73% of the total population

$z_1 = \mp 3$ & $z_2 = +3$.

$X = \mu + z\hat{\sigma}$.

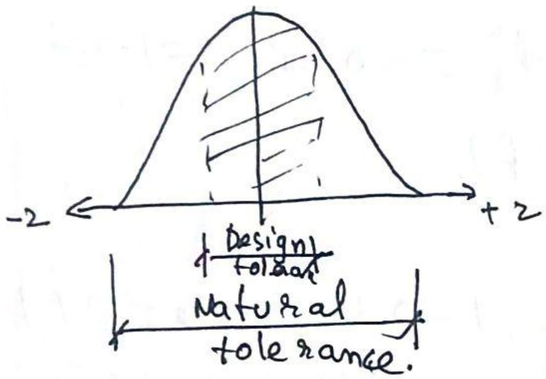
$\therefore X_1 = \mu + 3(\hat{\sigma})$ & $X_2 = \mu - 3(\hat{\sigma})$.

$\therefore$ Natural tolerances are $\pm 3\hat{\sigma}$

ign tolerances are specification limits, somewhat arbitarily by the designer. om considerations of the proper matching the two components & functioning of the ssembly.

The design tolerances can be achieved only when the manufacturing process is so selected that the natural tolerances are within the design tolerances.

The following observations are made.

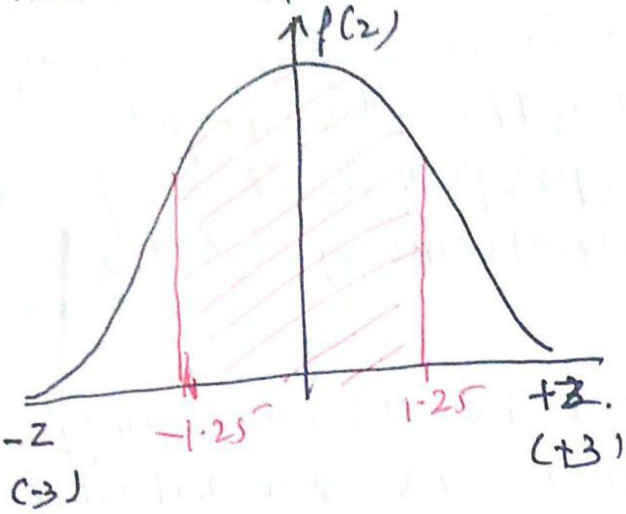


i) when the design tolerance is less than ($\pm 3\sigma$), the the percentage of rejected components is inevitable

ii) when the design tolerance is equal to ($\pm 3\sigma$) there is virtually no rejection. provided that the manufacturing process is centered.

iii) when the design tolerance is slightly greater than ($\pm 3\sigma$) there is no rejection, even if the manufacturing process is slightly off-centre..

2) It has been observed from a sample of 200 bearing bushes that the diameters are normally distributed with a mean of 30.010 & a standard deviation of 0.008 mm. The upper & lower limits for the internal are specified by the designer, are 30.02 & 30 mm respectively. Calculate the percentage of rejected bushes



$n = 200$

$\mu = 30.010$ // $x_1 = 30.02$ m

$\sigma = 0.008$ // $x_2 = 30.00$

$z = \dfrac{x_1 - \mu}{\sigma}$ //

$$z = \frac{30 - 30.010}{0.008} = -1.25$$

$$z = \frac{30.02 - 30.010}{0.008} = 1.25$$

Area of shaded = 2 (area bet^n^ $z=0$ & $z = 1.25$)

$= 2(0.3944)$

$= 0.788$

$\therefore$ % of rejected bushes $= (1 - 0.788) \times 100 = 21.12\%$

Q) The diameters of a bolt are normally distributed with a mean of 10.02 mm and a standard deviation of 0.01 mm. The design specification for the diameter are 10 ± 0.025 mm. Calculate the percentage of bolt likely to be rejected

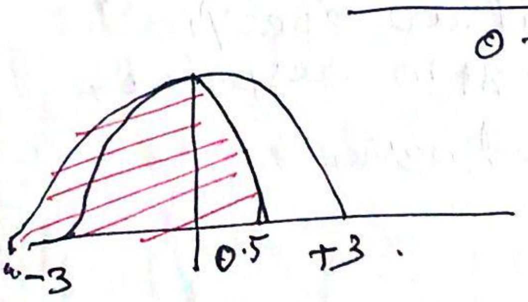
Ans). $\mu = 10.02$ $\quad x_1 = 10.025$

$\sigma = 0.01$ $\quad x_2 = 9.975$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{10.025 - 10.02}{0.01} = 0.5$$

(5×10^{-3})

$$z_2 = \frac{9.975 - 10.02}{0.01} = -4.5$$



Area from $z = 0$ to $z = 0.5$ = 0.1915

$\therefore$ Rejected area $= 0.5 - 0.1915$

$\therefore$ Rejection % $= 30.85\%$

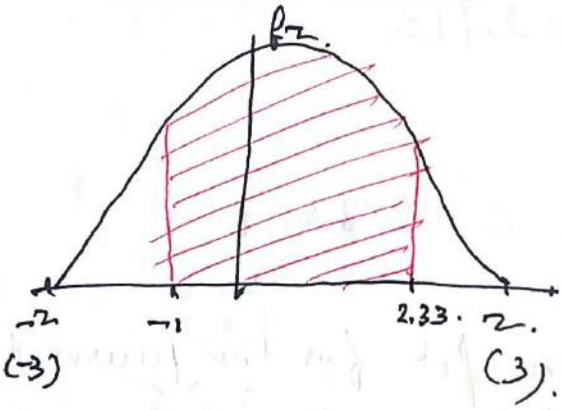
b) The tolerance specified by the designer for the diameter of transmission shaft is 25.000 ± 0.025 mm. The shafts are machined on three different machines. It was observed from the sample of shafts that the diameters are normally distributed with a standard deviation of 0.015 mm for each of the three machines. However, the mean diameter of shafts fabricated on the three machines is found to be 24.99 ~~& 25.01~~, 25 & 25.01 respectively. Determine the percentage of rejected shafts in each case & comment on the result.

Ans). we have

$x_1 = 25 - 0.025 = 24.975$

$x_2 = 25.000 + 0.025 = 25.025$

for Machine A.



$Z_1 = \frac{x_1 - \mu}{\sigma} = \frac{24.975 - 24.99}{0.015}$

$Z_1 = -1.$

$Z_2 = \frac{x_2 - \mu}{\sigma} = \frac{25.025 - 24.99}{0.015}$

$Z_2 = 2.33.$

Area between 0 to -1 = 0.3413

∴ ~~% rejection~~ on -ve = ~~0.6587~~.

Area between 0 to 2.33 = 0.4901.

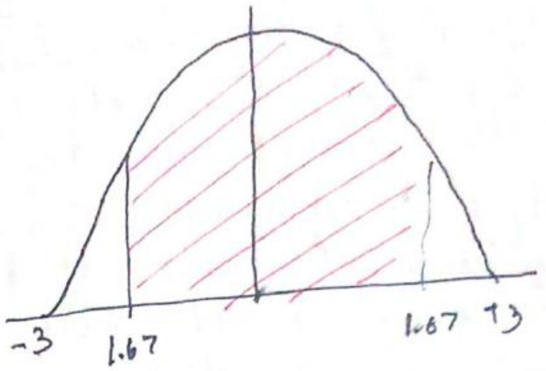
∴ Total area = 0.8314

∴ % rejection of Machine A = (1 − 0.8314) × 100
= 16.86%

for machine B.

$Z_1 = \frac{x_1 - \mu}{\sigma} = \frac{24.975 - 25}{0.015} = -1.67$

$Z_2 = \frac{25.025 - 25}{0.015} = 1.67.$



Area 0 to 1.67 = 0.4525

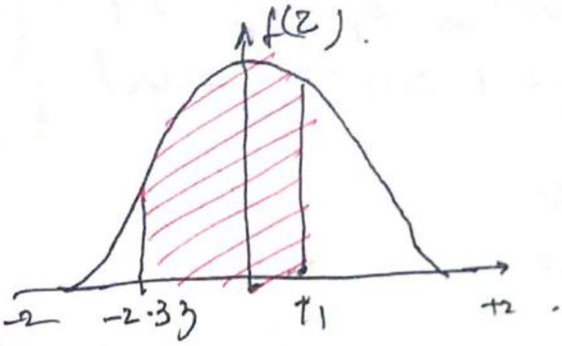
Total area = 2 × 0.4525

= 0.905

∴ % rejection = (1 − 0.905) × 100

= 9.5 %

Machine Z_1.



$$Z_1 = \frac{X_1 - \mu}{\sigma}$$

$$Z_1 = \frac{24.975 - 25.01}{0.015} = -2.33$$

$$Z_2 = \frac{25.025 - 25.01}{0.015} = +1.$$

Area from 0 to −2.33 = 0.4901

Area from 0 to +1 = 0.3413

Total area = 0.8314

% rejection = 1 − 0.8314 = 0.1686 × 100

= 16.86 %.

Q) The recommended class of transition fit for the journal and the bush of a hydrodynamic bearing is

Q) The recommended class of transition fit between the recess & the spigot of a rigid coupling is 60H6-j5. Assuming that the dimensions of the components are normally distributed, & that the specified tolerance is equal to natural tolerance, determine the probability of interference fit between the two components.

Q) The recommended class of fit for the journal & the bush of a hydrodynamic bearing is 40H6-e7. The dimensions of the bush are normally distributed & the natural tolerance is equal to design tolerance. From the considerations of hydrodynamic action & bearing stability, the maximum & minimum clearance are limited to 0.08 & 0.06 respectively. Determine the percentage of rejected assemblies.

Ans). Bush Population 1 – Bush.

40H6.

$40^{+0.016}$

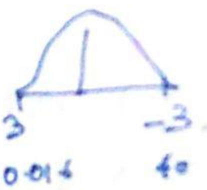
UL – 40.016.

LL – 40.000

$\therefore \mu_H = 40.008.$

for 40.016

$\hat{\sigma} = \frac{X-\mu}{Z}.$



$\hat{\sigma} = \frac{40.016 - 40.008}{3}$

$\hat{\sigma}_H = 2.67 \times 10^{-3}$ mm.

Population 2 – Journal.

e7 → es = -50, ei = -75

$40^{-0.050}_{-0.075}$

UL – 39.950

LL – 39.925

$\therefore \mu_J = 39.9375$

for 39.95.

$\hat{\sigma} = \frac{X-\mu}{Z}.$

$\hat{\sigma} = \frac{39.95 - 39.9375}{3}$

$\hat{\sigma}_J = 4.167 \times 10^{-3}$

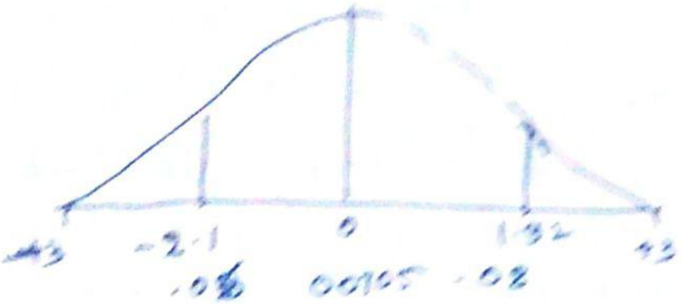
Population 3

clearance.

$\mu_c = \mu_H - \mu_J = 40.008 - 39.9375 = 0.0705$

$\sigma_c = \sqrt{(2.67 \times 10^{-3})^2 + (4.167 \times 10^{-3})^2}$

$\sigma_c = 4.95 \times 10^{-3}$



Area $Z = \frac{x-\mu}{\sigma}$

$Z = \frac{0.06 - .0705}{4.95 \times 10^{-3}}$

$\boxed{Z = -2.1}$

$Z = \frac{x-\mu}{\sigma}$

$= \frac{0.08 - 0.0705}{4.95 \times 10^{-3}}$

$= 1.92$

Area 0 to -2.1 = 0.4821

Area 0 to 1.92 = 0.4719

0.954

$\therefore$ Rejection $= (1 - 0.954) \times 100$

$= 4.6\%$

Reliability

A product is said to be reliable when it performs its intended function satisfactorily throughout its life.

According to international standards organization reliability is defined as ability of an item to perform a required function under stated conditions for a stated period of time.

In engineering design, reliability is expressed quantitatively as 0.9 or 90%. In such cases, reliability is defined as:

the probability that a product will perform required function under stated conditions for a stated period of time.

so reliability contains four basic elements

i) The reliability of the product is expressed as a probability

ii) The product is required to perform its intended function.

iii) The period during which the product has to perform the function is specified.

iv) The operating conditions under which the product has to function are specified.

eg: Ball bearing subjected to radial load of 5 kN, expected life 8000 Hr. Shaft rotates at a speed of 1450 rpm.

1- Manufacturers give a reliability of 90%.

2- It has to rotate freely with a load of 5 kN (1450 rpm)

3- It has to run for 8000 Hrs.

4- Lubrication, tolerances etc.

Product reliability & product quality are closely related to each others.

Reliability is described as quality main during useful life of the product.

Probabilistic approach to Design

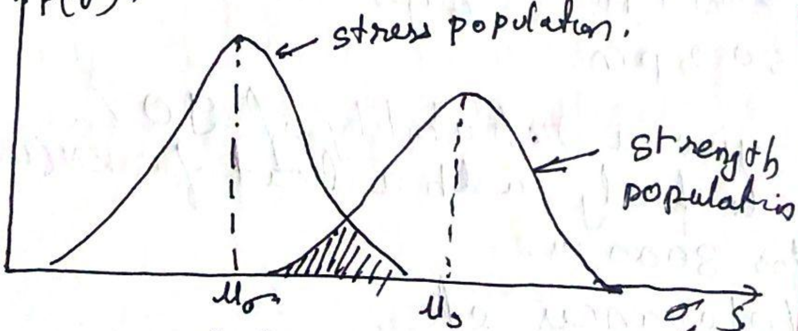
Its not possible to determine Reliability using the concept of factor of safety. Since reliability is a design parameter, it should be incorpor in the product at design stage. Factor of safety does not address reliability.

Probabilistic approach is a technique to design the component for a given magnitude of reliability, in this approach following assumptions are made.

i) The ultimate tensile strength or yield strength is not constant but subjected to statistical variation. The population of strength, denoted by S is under statistical control. It is normally distributed with a mean μ_s and standard deviation $\hat{\sigma}_s$.

ii) The stress induced in the component is not constant but subjected to statistical variation. The population of stress, denoted by σ is normall distributed, with a mean of μ_σ and standard deviation of $\hat{\sigma}_\sigma$

fig. 1



Normal distrebution of strength & stress population

→ The mean of strength population is more than the mean of stress population.

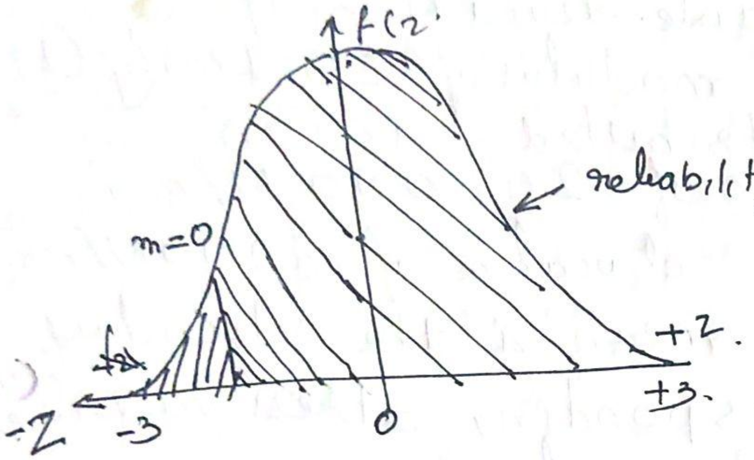
→ The forward tail of stress is overlapping the rear tail of strength & is the region of unreliability

→ some failure occurs in this region.

A third population of margin of safety is formed by subtracting population of stress from the population of the strength. The population of margin of safety is denoted by m. (12)

$$\mu_m = \mu_s - \mu_\sigma .$$

$$\& \quad \hat{\sigma}_m = \sqrt{(\hat{\sigma}_s)^2 + (\hat{\sigma}_\sigma)^2},$$



$$Z = \frac{m - \mu_m}{\hat{\sigma}_m}$$

when stress is equal to strength, margin of safety is zero & failure may occur.

$$\therefore m = 0.$$

$$Z_0 = \frac{0 - \mu_m}{\hat{\sigma}_m} = -\frac{\mu_m}{\hat{\sigma}_m}$$

Four basic ways to increase reliability

i) Increase the mean of strength population (μ_s). by using a better quality material for the component

ii) Decrease the mean of stress population (μ_σ) by increasing size of component

iii) Decrease the standard deviation ($\hat{\sigma}_\sigma$) of stress population by controlling manufacturing method.

iv) Decrease the standard deviation ($\hat{\sigma}_s$) of strength population by controlling quality of the incoming material.

All the above increases the cost of the component. Reliability is achieved by increasing the cost of the component. So high reliability can't be the criterion in all applications

Q) A tension rod is subjected to axial stress within elastic limit. According to Hooke's law.

$\sigma = E\epsilon$.

It has been observed that the strain (ϵ) in the tension rod is a normally distributed variable with a mean of 0.001 mm/mm and a standard ~~variable~~ deviation of 0.00007 mm/mm. The modulus of Elasticity (E) is also normally distributed random variable with a mean of 207000 N/mm² and a standard deviation of 6000 N/mm².

Determine the mean & the standard deviation of the corresponding stress variable σ. Comment on the analysis.

An). $\mu_\sigma = \mu_E \mu_\epsilon$.

we have $Z = xy$ then.

$$\hat{\sigma}_z = \sqrt{\mu_x^2(\hat{\sigma}_y)^2 + \mu_y^2(\hat{\sigma}_x)^2 + (\hat{\sigma}_x)^2(\hat{\sigma}_y)^2}$$

given $\mu_\epsilon = 0.001$ $\sigma_\epsilon = 0.00007$.

$\mu_E = 207000$ $\sigma_E = 6000$

$$\hat{\sigma}_\sigma = \sqrt{\mu_\epsilon^2(\hat{\sigma}_E)^2 + \mu_E^2(\hat{\sigma}_\epsilon)^2 + (\hat{\sigma}_E)^2(\hat{\sigma}_\epsilon)^2}$$

$$\sigma_\sigma = \sqrt{(0.001)^2(6000)^2 + (207000)^2(0.00007)^2 + (0.00007)^2(6000)^2}$$

$\sigma_\sigma = 15.69$ N/mm². — Ans.

$\mu_\sigma = \mu_\epsilon \mu_E = 0.001 \times 207000 = 207$ N/mm² — Ans.

we can predict the mean & the standard deviation of stress population

) A rod is subjected to pure uniaxial strain, which is given by.
$\epsilon = \frac{\delta}{l}$. It has been observed that the length (l). of the rod is a normally distributed random variable with a mean of 100 mm & a standard deviation of 0.5 mm. The deflection of the rod ~~axis~~ (δ) is also normally distributed random variable with a mean of 0.075 mm & a standard deviation of 0.005 mm.

Determine the mean & standard deviation of the corresponding strain variable. Comment on the analysis,

A). for length.

$\mu_l = 100$ mm.

$\sigma_l = 0.5$ mm.

for deflection.

$\mu_\delta = 0.075$

$\sigma_\delta = 0.005$

If $Z = \frac{x}{y}$.

$$\sigma_z = \frac{1}{\mu_y}\left[\frac{\mu_x^2(\hat{\sigma}_y)^2 + \mu_y^2(\hat{\sigma}_x)^2}{\mu_y^2 + (\hat{\sigma}_y)^2}\right]^{1/2}$$

$\epsilon = \frac{\delta}{l}$

$$\therefore \sigma_\epsilon = \frac{1}{\mu_l}\left[\frac{\mu_\delta^2(\hat{\sigma}_l)^2 + \mu_l^2(\hat{\sigma}_\delta)^2}{\mu_l^2 + (\hat{\sigma}_l)^2}\right]^{1/2}$$

$$= \frac{1}{100}\left[\frac{0.075^2(0.5)^2 + 100^2(0.005)^2}{100^2 + (0.5)^2}\right]^{1/2}$$

$$\hat{\sigma}_\epsilon = \cancel{0.0}\ 5 \times 10^{-5} \text{ mm/mm}$$

$$\mu_\epsilon = \frac{\mu_\delta}{\mu_l} = \frac{0.075}{100} = 7.5 \times 10^{-4}$$

we can predict σ_ϵ & μ_ϵ

Q) A beam of circular crossection is subjected to pure bending moment M and the bending stresses are given by the following equation

$$\sigma = \frac{32 M_b}{\pi d^3}$$

where d is the diameter of the beam. It has been observed that the diameter (d) of the beam is normally distributed random variable with a mean of 50 mm & a standard deviation of 0.125 mm. The bending moment (M_b) is also normally distributed random variable with a mean of 1750 N-m & a standard deviation of 150 N-m

Determine the mean standard deviation of the corresponding bending stress variable (σ).

Ans) Let $\frac{32}{\pi d^3} = Z$. $Z = \frac{\pi}{32} d^3$

for shaft
$\mu_d = 50$
$\sigma_d = 0.125$

$\therefore \sigma = \frac{M_b}{Z}$

$\mu_z =$ If $Z = x^3$

$$\mu_z = \left[\mu_x^3 + 3\mu_x (\hat{\sigma}_x)^2\right] \frac{\pi}{32}$$

$$\mu_z = \left[\mu_d^3 + 3\mu_d (0.125)^2\right] \frac{\pi}{32}$$

$$\mu_z = \left[(50)^3 + 3(50)(0.125)^2\right] \frac{\pi}{32}$$

$$\mu_z = \cancel{1227}\ 12273.67 \text{ N mm}^3$$

If $Z = x^3$

$$\hat{\sigma}_z = 3\mu_x^2(\hat{\sigma}_x) + 3(\hat{\sigma}_x)^3$$

In the problem.

$$\hat{\sigma}_z = \left[3\mu_d^2(\hat{\sigma}_d) + 3(\hat{\sigma}_d)^3\right] \frac{\pi}{32}$$

$$\sigma = \frac{32\, M_b}{\pi d^3}$$

$$\sigma = \left(\frac{32}{\pi}\right) \frac{M}{d^3}.$$

let $z = \frac{\pi}{32} d^3.$

$$\mu_z = [\mu_x^3 + 3\mu_x (\hat{\sigma}_x)^2] \frac{\pi}{32}$$

He $\mu_x = 50$ m $\quad \sigma_x = 0.125.$

$$\mu_z = \frac{\pi}{32} [\mu_x^3 + 3\mu_x (\hat{\sigma}_x)^2].$$

$$\mu_z = \frac{\pi}{32}[(50)^3 + 3 \times 50 (0.125)^2].$$

$$\boxed{\mu_z = 12273.7 \text{ mm}^3}$$

$$\hat{\sigma}_z = [3\mu_x^2 (\hat{\sigma}_x) + 3(\hat{\sigma}_x)^2.]$$

$$\sigma_z = [3(50)^2 (0.125) + 3(0.125)^2] \frac{\pi}{32}$$

$$\boxed{\sigma_z = 92.05 \text{ mm}^3}$$

for $\sigma = \frac{32 M}{\pi d^3} = \frac{M}{z}.$

& Now we create a ~~the~~ fourth population

$$\sigma = \frac{M}{z}.$$

where $\mu_f = \frac{\mu_m}{\mu_z}.$

$$\mu_f = \frac{1750 \times 10^3}{12273.7} = 142.6 \text{ N/mm}^2.$$

$$\sigma_f = \frac{1}{\mu_z}\left[\frac{\mu_m^2 (\hat{\sigma}_z)^2 + \mu_z^2 (\hat{\sigma}_m)^2}{\mu_z^2 + (\hat{\sigma}_z)^2}\right]^{1/2}$$

$$= \frac{1}{12273.7}\left[\frac{(1750 \times 10^3)^2 (92.05)^2 + (12273.7)^2 (150 \times 10^3)^2}{(12273.7)^2 + (92.05)^2}\right]^{1/2}$$

$$= 12.27 \text{ N/mm}^2$$